

# Efficient Decentralized Coordination of Large-scale Plug-in Electric Vehicle Charging

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# Motivation

- The electrical power drawn by plug-in electric vehicle (PEV) chargers will (eventually?) begin to impact the grid.
- At the system-wide level, control objectives tend to focus on filling the overnight valley in background demand.
- At the distribution level, proposed control strategies address:
  - Transformer overloads
  - Loss minimization
  - Voltage degradation
  - Tap-change minimization
- Few control strategies also take into account the effects of charging on battery health.

# Goals

- A decentralized approach to scheduling PEV charging that considers trade-offs between:
  - Energy price
  - Battery degradation
  - Distribution network effects
- The resulting collection of PEV charging strategies should be efficient (socially optimal).
- Convergence should only require a few iterations.

# Formulation

- PEV population:  $\mathcal{N} \equiv \{1, \dots, N\}$ .
- Horizon:  $\mathcal{T} \equiv \{0, \dots, T - 1\}$ .
- Admissible charging strategies:

$$u_{nt} \geq 0, \quad t \in \mathcal{T}$$
$$\|\mathbf{u}_n\|_1 \equiv \sum_{t \in \mathcal{T}} u_{nt} \leq \Gamma_n$$

where  $\Gamma_n$  is the energy capacity of the  $n$ -th PEV.

- The set of admissible charging controls is denoted  $\mathcal{U}_n$ .

# Demand charge

- Distribution-level impacts are largely a consequence of coincident high charger power demand  $u_{nt}$ .
- Undesirable effects can be minimized by encouraging lower power levels,

$$Cost_{demand,nt} = g_{demand,nt}(u_{nt})$$

where  $g_{demand,nt}(\cdot)$  is a strictly increasing function.

# Battery degradation cost

Experimentation with  $\text{LiFePO}_4$  lithium-ion batteries gave an (empirical) degradation model:

$$\mathfrak{d}_{\text{cell}}(I, V) = \beta_1 + \beta_2 I + \beta_3 V + \beta_4 I^2 + \beta_5 V^2 + \beta_6 IV + \beta_7 V^3$$

relating energy capacity loss per second (in  $\text{Amp} \times \text{Hour} \times \text{Sec}^{-1}$ ) to charging current  $I$  and voltage  $V$ .

- Degradation cost:

$$\mathfrak{g}_{\text{cell}}(I, V) = P_{\text{cell}} \Delta TV \mathfrak{d}_{\text{cell}}(I, V)$$

where  $P_{\text{cell}}$  is the price (\$/Wh) of battery cell capacity.

- Over the useable state of charge (SoC) range,  $V \approx V_{\text{nom}}$ .
- Battery degradation cost can be expressed as:

$$\begin{aligned} \text{Cost}_{\text{degrad}, nt} &= g_{\text{cell}, n}(u_{nt}) = M_n \mathfrak{g}_{\text{cell}}\left(\frac{10^3 u_{nt}}{M_n V_{\text{nom}}}, V_{\text{nom}}\right) \\ &= a_n u_{nt}^2 + b_n u_{nt} + c_n \end{aligned}$$

# Centralized formulation

System cost:

$$J(\mathbf{u}) \triangleq \sum_{t \in \mathcal{T}} \left\{ c \left( d_t + \sum_{n \in \mathcal{N}} u_{nt} \right) + \sum_{n \in \mathcal{N}} g_{nt}(u_{nt}) \right\} - \sum_{n \in \mathcal{N}} \left\{ h_n(\|\mathbf{u}_n\|_1) \right\}$$

where:

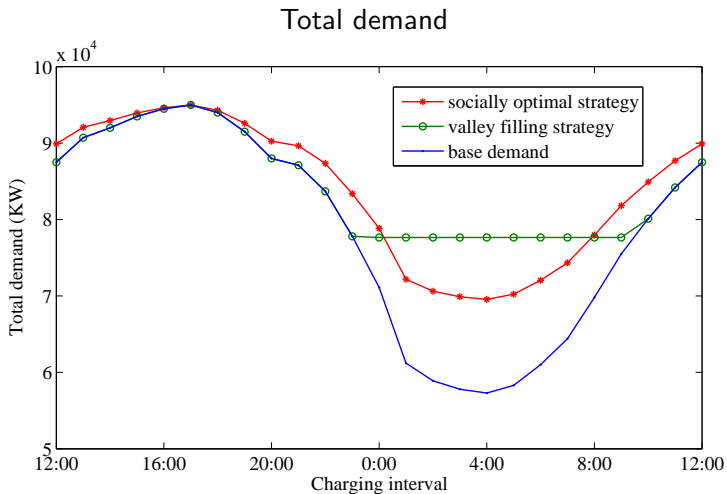
- $\mathbf{u}_n \in \mathcal{U}_n$  for all  $n \in \mathcal{N}$ .
- $c(\cdot)$  gives the generation cost with respect to the total demand  $d_t + \sum_{n \in \mathcal{N}} u_{nt}$ , and  $d_t$  denotes the aggregate inelastic base demand at time  $t$ .
- $g_{nt}(u_{nt}) = g_{demand,nt}(u_{nt}) + g_{cell,n}(u_{nt})$  captures the demand charge and battery degradation cost of the  $n$ -th PEV.
- $h_n(\|\mathbf{u}_n\|_1)$  denotes the benefit function of the  $n$ -th PEV with respect to the total energy delivered over the charging horizon, with:

$$h_n(\|\mathbf{u}_n\|_1) = -\delta_n(\|\mathbf{u}_n\|_1 - \Gamma_n)^2$$

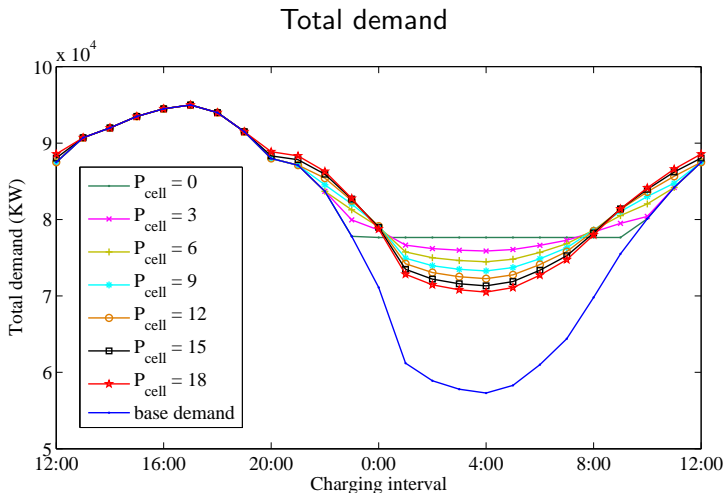
# Assumptions

- (A1) The generation cost function  $c(\cdot)$  is monotonically increasing, strictly convex and differentiable.
- (A2) The combined demand charge and battery degradation cost  $g_{nt}(\cdot)$ , for all  $n \in \mathcal{N}$ ,  $t \in \mathcal{T}$ , is monotonically increasing, strictly convex and differentiable.
- (A3) The benefit function  $h_n(\omega)$  is differentiable, increasing and strictly concave on  $0 \leq \omega \leq \Gamma_n$ .

# Example

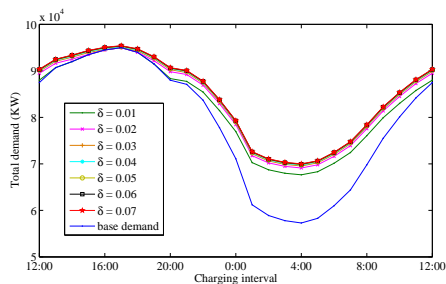


# Example - varying $P_{cell}$

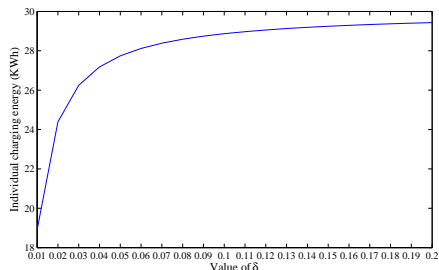


# Example - varying terminal penalty, $\delta_n$

Total demand



Delivered energy



# Decentralized charging coordination

- (S1) Each PEV autonomously determines its optimal charging strategy with respect to a given electricity price profile  $\mathbf{p} \equiv (p_t, t \in \mathcal{T})$ . This optimal strategy takes into account the trade-off between the electricity cost and local (demand and battery degradation) costs over the entire charging horizon.
- (S2) The electricity price profile  $\mathbf{p}$  is updated to reflect the latest charging strategies determined by the PEV population in (S1).
- (S3) Steps (S1) and (S2) are repeated until the change in the price profile at (S2) is negligible.

Using an appropriate individual cost function and price update mechanism, (S1)-(S3) is convergent and achieves the socially optimal (centralized) solution.

# Individual cost function

$$J_n(\mathbf{u}_n; \mathbf{p}) = \sum_{t \in \mathcal{T}} p_t u_{nt} + g_{nt}(u_{nt}) - h_n \sum_{t \in \mathcal{T}} u_{nt}$$

- Cost is composed of energy cost, local (demand and battery degradation) cost, and the benefit derived from the total delivered energy.
- The optimal charging strategy of the  $n$ -th PEV, with respect to  $\mathbf{p}$ :

$$\mathbf{u}_n^*(\mathbf{p}) = \operatorname{argmin}_{\mathbf{u}_n \in \mathcal{U}_n} J_n(\mathbf{u}_n; \mathbf{p})$$

- This optimal response has the form:

$$u_{nt}(\mathbf{p}, A_n) = \max \{0, [g'_{nt}]^{-1}(A_n - p_t)\}, \quad t \in \mathcal{T}$$

for some  $A_n$ , where  $g'_{nt}$  is the derivative of  $g_{nt}$ , and  $[g'_{nt}]^{-1}$  denotes the corresponding inverse function.

# Price profile update mechanism

Let

$$p_t^+(\mathbf{p}) = p_t + \eta \sum_{n \in \mathcal{N}} (d_t + u_{nt}^*(\mathbf{p})) - p_t, \quad t \in \mathcal{T}$$

where  $\eta > 0$  is a fixed parameter, and  $\mathbf{u}_n^*(\mathbf{p})$  is the optimal charging strategy for the  $n$ -th PEV with respect to  $\mathbf{p}$ .

- The price update mechanism can be expressed as,

$$\mathbf{p}^+(\mathbf{p}) = (1 - \eta)\mathbf{p} + \eta\mathcal{P}(\mathbf{p})$$

- This has the form of the Krasnoselskij iteration, and is therefore guaranteed to converge to a fixed point of  $\mathcal{P}(\cdot)$  for any  $\eta \in (0, 1)$  if  $\mathcal{P}(\cdot)$  is non-expansive.

# Main results

## Lemma:

$$\|\mathbf{u}_n^*(\mathbf{p}) - \mathbf{u}_n^*(\mathbf{q})\|_1 \leq 2\nu \|\mathbf{p} - \mathbf{q}\|_1$$

where  $\nu$  is the maximum over the Lipschitz constants of  $[g_{nt}]^{-1}(\cdot)$ .

**Theorem:** The decentralized algorithm converges to the efficient (centralized) solution  $\mathbf{u}^{**}$ . For any  $\varepsilon > 0$ , convergence  $\|\mathbf{p} - \mathbf{p}^{**}\| \leq \varepsilon$  is guaranteed in no more than  $K(\varepsilon)$  iterations.

- $K(\varepsilon)$  involves the price update parameter  $\eta$ , number of vehicles  $N$ , time horizon  $T$ , Lipschitz constant for  $c(\cdot)$  and maximum Lipschitz constant over  $[g_{nt}]^{-1}(\cdot)$  (given by  $\nu$ ).
- The proof establishes that

$$\|\mathbf{p}^+ - \mathbf{q}^+\|_1 < \|\mathbf{p} - \mathbf{q}\|_1$$

so the price update operator  $\mathbf{p}^+(\mathbf{p})$  is a contraction map.

# Consensus-based solution

- Assume the generation cost  $c(\cdot)$  is quadratic.
- The following completely distributed process achieves exactly the same outcome as the earlier iterative strategy.

(S1) Each PEV autonomously determines its optimal charging strategy  $\mathbf{u}_n^*(\mathbf{p})$ . It then computes its estimate of the updated price:

$$p_{nt}^+(\mathbf{p}) = p_t + \eta \left( c(d_t) + Nu_{nt}^*(\mathbf{p}) - p_t \right), \quad t \in \mathcal{T}$$

(S2) PEVs exchange their price estimates  $\mathbf{p}_n^+(\mathbf{p})$  with neighbours in an average consensus process to obtain the updated price profile:

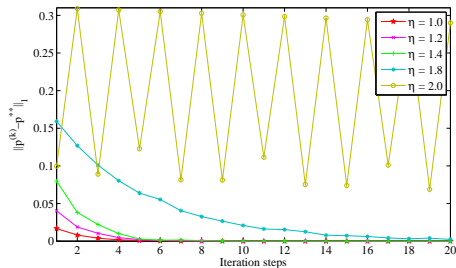
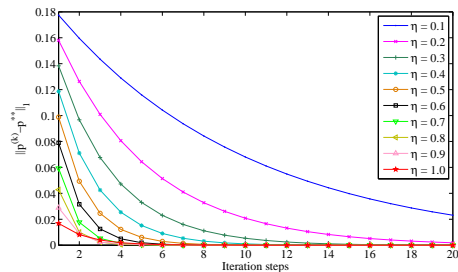
$$\mathbf{p}^+(\mathbf{p}) = \frac{1}{N} \sum_{n \in \mathcal{N}} \mathbf{p}_n^+(\mathbf{p})$$

(S3) Steps (S1) and (S2) are repeated until the change in the price profile at (S2) is negligible.

# Illustration - convergence

Evolution of  $\|\mathbf{p}^{(k)} - \mathbf{p}^{**}\|_1$  for various values of the price update parameter  $\eta$ .

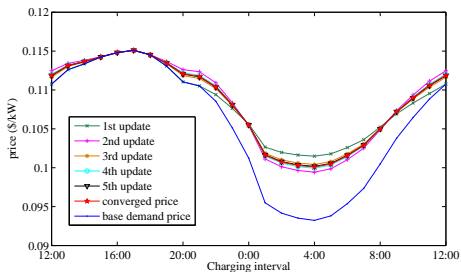
- Convergence is guaranteed for  $0 < \eta < 1.017$ .



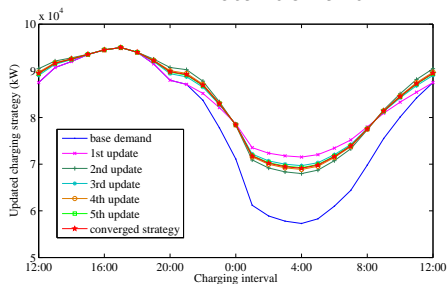
# Illustration - algorithm updates

Price update parameter  $\eta = 1$ .

Price



Total demand



# Conclusions

- A price-based decentralized strategy has been developed for coordinating the charging of a large population of PEVs.
- PEVs minimize a cost function that captures the trade-off between:
  - Cost of energy.
  - Costs associated with battery degradation.
  - High charging demand.
- A decentralized iterative scheme converges to the unique efficient collection of charging strategies.
  - Average consensus can be used to completely distribute this process.

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